A piece of cake

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| Year levelStrand(s)Lesson lengthCD Code | Year 10Measurement60 mins[AC9M10M01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/content-description?subject-identifier=MATMATY10&content-description-code=AC9M10M01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick), [AC9M10M05](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/content-description?subject-identifier=MATMATY10&content-description-code=AC9M10M05&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| Lesson summary | In this lesson, students build on their skills in finding volumes and surface areas of prisms and cylinders to design a multi-level celebration cake for a party. They use mathematical modelling to vary the size and shape of the component cakes and to calculate how much frosting is needed. |
| Learning intention | We will interpret practical modelling problems involving volume and surface area. We are learning how to use decomposition to find volumes and surface areas of composite solids comprising prisms or cylinders. |
| Success criteria | By the end of this lesson, students can:interpret a practical modelling problem, applying their knowledge of 3D solids to meet a design briefuse or build a mathematical model in a spreadsheet to generate a set of dimensions for a composite solid to meet a minimum volume criterion find the volume and surface area of a composite solid.  |
| Why are we learning about this? | Surface area and volume have many real-world applications, such as calculating how much paint is needed to paint a house or how much chlorine has to be added to a swimming pool. In manufacturing and engineering processes, a detailed understanding of volume and surface area calculations is essential and as most objects are not ‘known solids’, an ability to decompose into separate solids is required.  |
| Prerequisite student knowledge and language | Prior to this lesson, it is assumed that students have knowledge of:volume and surface area of cylinders and prismsarea of composite shapessome understanding of basic spreadsheet functionality including performing calculations using symbols such as = \* / and ^. |
| **Resources** | Teacher’s slides (PowerPoint)A piece of cake worksheet (Word)Excel model (Excel)Exit ticket (Word)Calculators, access to computers or laptops |

Curriculum information

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| Achievement standard | Students [solve measurement problems involving surface area and volume of composite objects](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10?view=quick&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0) and [use mathematical modelling to solve practical problems involving  proportion and scaling, evaluating and modifying models, and reporting assumptions, methods and findings.](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10?view=quick&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0) |
| Content description(s) | Students solve problems involving the surface area and volume of composite objects using appropriate units. [AC9M10M01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/content-description?subject-identifier=MATMATY10&content-description-code=AC9M10M01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)Students use mathematical modelling to solve practical problems involving proportion and scaling of objects; formulate problems and interpret solutions in terms of the situation; evaluate and modify models as necessary, and report assumptions, methods and findings. [AC9M10M05](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/content-description?subject-identifier=MATMATY10&content-description-code=AC9M10M05&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| General capabilitiesCross-curriculum priority | Numeracy:* Understanding geometric properties ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M01&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Proportional thinking ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M05&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNPrT&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Critical and Creative Thinking:* Interpret concepts and problems ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M01&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Draw conclusions and provide reasons ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10A04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=1&sub-element-code=CCTANAB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Consider alternatives ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10A04&general-capability-code=CCT&element-code=CCTGEN&sub-element-index=0&sub-element-code=CCTGENB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Evaluate actions and outcomes ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M05&general-capability-code=CCT&element-code=CCTANA&sub-element-index=2&sub-element-code=CCTANAC&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Identify, process and evaluate information ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M05&general-capability-code=CCT&element-code=CCTINQ&sub-element-index=0&sub-element-code=CCTINQB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Digital Literacy:Select and operate tools ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/digital-literacy/slideout?code=DLMOC6&element=3&sub-element=2)) |
| Areas of challenge | Some students may have trouble decomposing a composite solid – use physical and virtual manipulatives such as stacked cake tins or food boxes to demonstrate how composite solids can be decomposed into their constituent parts.Students may experience difficulties finding volumes – display key formulae ($V=lwh, V=Ah $and $V=πr^{2}h$) on the whiteboard, ask students to identify the meaning of each pronumeral and demonstrate on a diagram where they appear in each solid.Some students may make errors calculating surface area and miss or double-count faces, include ‘hidden’ faces or the bottom of the cake or forget to include the filling. They may wish to find the area of the top of each cake, which is partly covered by the smaller cake above. Encourage students to carefully mentally ‘walk around their cake’ marking off each face as they find its area and to think about how cakes can be mentally ‘moved’ to complete odd shapes and simplify calculations. Have students design a cake that matches the design in the samples in the Excel model so they can a correctly calculate surface area in the model, which they can then use to check their manual calculations. Students may have gaps in spreadsheet skills/knowledge – explicitly demonstrate and explain the key features and operation of spreadsheets. Use the provided Excel model and encourage students to interrogate formulae in cells to understand how the calculations have been derived. For students creating their own models, encourage use of online help videos. |
| Strategies | [Differentiated teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/differentiated-teaching/)[Mathematics investigation](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/mathematics-investigation/) |

Lesson structure

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| Learning hook10 mins | Use the teacher’s slides to complement the teaching of this lesson. These and other resources can be accessed through the online lesson in the ‘What you need’ section.**Warm-up activity**This activity allows students to access prior knowledge and practise using formulae to find the volumes of prisms and cylinders. (Slide 2) Show the image of the two pieces of cake and ask students to vote on which is bigger. Tally the results on the whiteboard. Ask the students:* ‘How can we confirm which answer is correct?’ Students should identify that they would need to calculate the volume of each piece of cake.
* ‘What information do we need to know?’ Students should identify the variables of height, width and length of the rainbow cake; height and radius of the chocolate cake; and the angle at the centre (or fraction of the whole cake).

(Slide 3) Provide the relevant information by annotating the whiteboard. Ask students to pair up and calculate the volumes of the two pieces of cake. If necessary, assist students by providing the formulae for the volume of a rectangular prism: $V=lwh,$ area of a sector $A=\frac{θ}{360}πr^{2}$; and volume of a prism $V=Ah$.Review the students’ answers.Answer: The chocolate cake is bigger, as it has a volume of approximately $134cm^{3}$ versus $120cm^{3}$ for the rainbow cake. Rainbow cake: $V=6×4×5=120cm^{3}$Chocolate cake: $A=\frac{40}{360}π×8^{2}≈22.340cm^{2} $and $V=22.340×6≈134cm^{3}$ |
| Explore45 mins | **Introducing the task**(Slide 4) Present learning intention and success criteria and ask students if they have ever been to a wedding and what they know about wedding cakes. Ask what kind of birthday cakes they have seen, at their own or relatives’ birthday parties, in shops or online. Collate their responses on the whiteboard. (Slides 5 and 6) Show the images to expand on the answers, which may include:* Wedding cakes often have many layers (three is common, but more layers are possible). The most common shape is round (cylindrical) but square-based cakes are also reasonably common.
* Birthday cakes often only have one layer, but can be in very complex (for example, pirate ships, ballet dancer, football, dinosaur).

(Slides 7–11) Explain the task. Students are to design a celebration cake (individually or in pairs).* Choose an occasion (wedding, birthday, baby shower …)
* Choose the number of party guests who will eat your cake (20 to 150)
* Your cake design must be a composite solid – made up of at least two separate solids – for example, a stack of cylinders, rectangular prisms or a more complex shape comprising different solids.
* Use the portion-size guide to work out the minimum volume of a portion and hence, the volume your cake must have.
* Use a spreadsheet model to vary the dimensions of the individual cakes to determine their ideal sizes.
* Calculate how many 600 mL tubs of frosting you will need to cover your cake.
* Complete the written report explaining your calculations.

**Additional information and guidance** (Slide 8) The portion-size guide gives the dimensions of an appropriate portion-size for a piece of cake for one person. Students can use this to find the volume of cake needed for one person: $V=lwh=12×3×5=180cm^{3}$.(Slides 9–1) Use these to scaffold the task. They provide guidance on how to find volume by decomposing their cake into its composite solids. They show how to calculate surface area by identifying and finding the area of each face, paying attention to hidden faces and partially covered faces, and considering the layers of frosting within each individual cake.**Modelling the problem**The Piece of cake model (Excel) illustrates the minimum volume, actual volume, surface area and frosting tubs calculations for two sample cakes: a two-cake cylindrical cake and a three-cake square-based cake. Teachers can distribute this model to students for them to amend or ask students to build their own model.Demonstrate the key features and assumptions in the model and key Excel conventions, including:* how cell references are used in Excel when performing calculations
* clearly indicating input cells using shading (blue in this model)
* using ‘=’ when requiring Excel to complete a calculation
* using ‘\*’ and ‘/’ respectively for multiplication and division operations
* using ‘^’ for powers/exponents
* using the command ‘PI()’ for $π$
* using the command ‘ROUNDUP’ in the calculation of the number of tubs.

**Independent work**Distribute the Piece of cake worksheet and the Piece of cake model (Excel). Monitor students as they proceed through the task.Encourage students to see the value and efficiency in using the Excel model to test multiple different sizes of cakes to meet the minimum volume requirement.Once students have identified their cake dimensions and used their model to calculate the volume, surface area and number of tubs of frosting required, they should reproduce the calculations on the worksheet, explaining their thinking and calculations.**Differentiation (extension)**:Encourage use of a mix of solid shapes, other prisms such as triangular or trapezoidal prisms, shapes described in the Australian Curriculum, including cones, pyramids, spheres and hemispheres. Challenge students to build their own model from scratch. Prepare and scale-up recipes and prepare a budget for making their cake.**Differentiation (mid-level)**: Encourage students to use the pre-populated model, but further develop it by adding an additional cake or cakes, using the copy function to copy key formulae, ensuring that totals are updated. Challenge students to evaluate the reasonableness of their dimensions from practical and aesthetic perspectives. Have students present their cake design and calculations to their peers in small groups, explaining and justifying their design decisions and calculations.**Differentiation** **(enable):** Encourage use of a cake design that matches the two samples in the Excel model or single layer (no filling) cakes. Scaffold the worksheet by providing diagrams. Use physical or virtual manipulatives including nets to assist in decomposition. Run the activity as a paired task. Provide dimensions of common cake-tin sizes to help focus decision-making. |
| Summary and reflection5 mins | Show slide 12 and ask for students to contribute their answers to the questions in pairs (and then share with the class):* What did I learn today?
* How did I approach the problem?
* What was tricky today?
* What do I need to remember?

Show Slide 13 and recap the key learnings from the lesson that have not been covered in the previous reflection activity.* **Composite solids:** split the composite solid into solids you know; know your formulae; and find the volume of each separate solid.
* **Be careful:** in surface area problems, some sides are hidden and shouldn’t be counted. ‘Move around the shape’ and find each area in turn. Volume is measured in cm3 and surface area in cm2.
* **Mathematical models:** can be used to solve problems, by changing a range of variables.
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| Assessment | Opportunities for assessment are provided below and can be downloaded from the ‘What you need’ section of the online version of the lesson plan.Review students’ worksheets and provide feedback.Distribute and collect the exit ticket. Note that there are two identical exit tickets per A4 page. |