Pythagoras investigations and applications

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| Year levelStrand(s)Lesson lengthCD Code | Year 8Measurement / Number60 mins[AC9M8M06,](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8M06&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) [AC9M8N01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8N01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| Lesson summary | This lesson is one in a series of lessons on Pythagoras theorem. Students work in small groups to investigate the application of Pythagoras’ theorem to problems in a practical or theoretical context. They apply the ‘context > representation > exploration > interpretation > investigation’ cycle to questions of interest or problems in a context involving right-angled triangles and lengths and communicate their findings to others. |
| Learning intention | * We are learning to identify information from a context relevant to the formulation of a problem.
* We will represent problems involving right-angled triangles and lengths using not to scale diagrams.
* We are learning to investigate solutions to a class of related problems involving Pythagoras’ theorem by systematically varying values.
* We will communicate and explain methods, reasoning and findings to others.
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| Success criteria | By the end of this lesson, students can:* identify relevant information in situations where Pythagoras’ theorem is a useful problem-solving tool
* represent this information on a suitable not-to-scale diagram
* systematically vary values to solve a class of related problems using Pythagoras’ theorem
* communicate and explain findings to others.
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| Why are we learning about this? | Interpreting information in context, recognising and using relevant mathematics, and systematically exploring solutions by varying values are key mathematical investigation skills. It is also important to be able to communicate and explain findings to others. |
| **Prerequisite student knowledge and language** | It is expected that students have familiarity with:* spatial terms including, side, right-angle, triangle, opposite, hypotenuse, square and their representation and labelling in diagrams
* measurement terms, including: 90 degrees, unit, cm, m, square cm, square m, length, area
* the number terms and notations for: square, square root, approximation, decimals and rounding.

Note: For the circle aspect of exploration Context 3, students would also need to be familiar with radius, diameter, semi-circle and formulas for the area of a circle. |
| **Resources** | * Teacher’s slides (PowerPoint)
* Scientific calculator
* Access to an online Pythagoras calculator and laptop or computer
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Curriculum information

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| Achievement standard | By the end of Year 8, students recognise irrational numbers and terminating or recurring decimals. Students use Pythagoras’ theorem to solve measurement problems involving unknown lengths of right-angled triangles.  |
| Content description(s) | * Students recognise irrational numbers in applied contexts, including square roots and π**.** [AC9M8N01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8N01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)
* Students use Pythagoras’ theorem to solve problems involving the side lengths of right-angled triangles**.** [AC9M8M06](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8M06&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)
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| General capabilitiesCross-curriculum priority | **General capabilities**Numeracy* Understanding geometric properties ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M06&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Understanding units of measurement ([Level 10](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M06&general-capability-code=N&element-code=NM&sub-element-index=1&sub-element-code=NMUuM&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Multiplicative strategies ([Level 9](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M06&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNMuS&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Critical and creative thinking* Interpret concepts and problems ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* consider alternatives ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=CCT&element-code=CCTGEN&sub-element-index=0&sub-element-code=CCTGENB&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Digital literacy* Select and operate tools ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=DL&element-code=DLMO&sub-element-index=0&sub-element-code=DLMOC&elaboration-code=AC9M8N01_E2&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
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| Areas of challenge | Some students may:* find difficulty identifying key information from text
* misrepresent key information on a diagram
* not understand the notion of a variable in terms of systematically exploring a context.
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| Strategies | [Mathematics investigation](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/mathematics-investigation/)[Questioning](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/questioning/)[Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/) |

Lesson structure

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| Learning hook15 mins | This activity is designed to engage students in exploring how Pythagoras’ theorem can be used to assist in solving problems involving right-angled triangles and lengths. The activity can be carried out in small groups of 2–3 students or individually. It also serves to refresh aspects of approaches students will need to use in their mathematical investigations.**Introduction**To begin, pose the following questions (slide 4): * You have an equilateral triangle with side lengths 20 cm.
* What is the perimeter of the triangle?
* What is the area of the triangle?

**Differentiation** (enable): Use the [Questioning](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/questioning/) strategy to prompt student thinking: How is the area of a triangle calculated? Draw a diagram of the triangle and label its base and height (altitude). What is the base of the triangle? What is its height?**Differentiation** (extend): Prompt students with, ‘if the side of the equilateral triangle is of length *a*, what are the formulas for the perimeter and area of the triangle?’* Explain that a key step to finding the area is to draw a diagram and add in a new line segment corresponding to the height (altitude) of the triangle.
* This splits the equilateral triangle into two identical right-angled triangles on either side of a (vertical) line of symmetry. Pythagoras theorem can then be applied to either of these triangles to find the height of the equilateral triangle, followed by the calculation for the area of a triangle using *Area* = ½ × *base* × *height*.
* Note: It’s not necessary to involve simplification of surds (square roots) at this stage, intermediate working using unevaluated square roots, and a decimal numerical approximation for a final answer is sufficient.
* Show and discuss the solution on slides 5 and 6.

**Learning hook*** In this part of the lesson, explain to students that they will now work in pairs or small groups to investigate one of three real-world contexts (or scenarios if you prefer) where they will use Pythagoras’ theorem, right-angled triangles and lengths to model a problem and systematically explore the context using variables.
* The students will then communicate back to the class their investigative process as well as their findings. The class will be able to ask clarifying questions at each stage and provide feedback on the presentation of the investigation. The three contexts provide opportunity for differentiation and student agency.

**Differentiation** (all abilities): Students can select all or part of the related questions of the given contexts to investigate; this allows all students to work at a suitable level where scaffolding and direction can be given as they progress. You may wish to assign students to groups and to particular context; or for some students, you may allow them to choose their own context that interests and engages them. **The investigation cycle**Show slide 7 and explain that they will use the Investigation cycle to help them do this. Discuss its four components and that by using an investigative process (which you can use out of maths class too) that it allows students to logically investigate their problem contexts.Broadly speaking explain that the four elements of the cycle include: * identifying the problem that needs to be investigated
* translating that problem and context in mathematical terms using symbols, diagrams, equations, graphs etc.
* exploring and analysing the problem by changing or manipulating the variables and finding the relationship between the variables to really understand the problem
* interpreting what you have found out in your maths investigation and then communicate what you have found out to others.

Now translate how a real-world context involving ‘triangles and Pythagoras’ can be inserted into the cycle. This connects the abstract thought to tangible contexts.The steps become: * **Step 1:** Understand the context for investigation and the corresponding related questions of interest and/or problems to solve.
* **Step 2:** Represent the situation using suitable right-angled triangle diagrams.
* **Step 3:** Select relevant variables (like length or area), and systematically vary these variables that allow you to apply Pythagoras’ theorem to answer questions of interest and/or solve related problems.
* **Step 4:** Summarise, interpret and communicate findings.
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| Explore25 mins | **Investigation**Now introduce the three possible contexts briefly outlining the related investigative questions that correspond with each real-world context. As mentioned above, students are to select only **1 of the 3** contexts.**Context 1: making a table** * (Slide 8) Right-angled triangles and related lengths are common in carpentry, woodwork and furniture design, for example, supports for shelves and rectangular tables with struts, especially as it’s important for perpendicular and horizontal orientations and keeping things square and rigid.
* Allow students to investigate using real examples of strutted tables and coffee tables from print or online furniture catalogues, brochures or websites, for example, The Strut collection, featuring tables on Blu Dot website.

Investigative questions* What will be the lengths of the struts required for different placements from the edge of the table in this design style?
* What are suitable dimensions for some different types of tables using this design style, for example, coffee table, dinner table, drink table, work desk?

**Context 2: modelling a cable stay bridge*** (Slides 10) Explain that cable stay bridges are one of the main types of bridge design and are commonly used for long bridges. The longest cable stay bridge in the world connects mainland China with Hong Kong: Hong Kong-Zhuhai-Macau Bridge
* Point out the diagonal cables that connect to vertical towers with the horizontal road platform.
* Slide 11 shows several different designs for cable stay bridges.

Investigative problem* Develop a model for each of these designs by assuming the distance between the two vertical ‘towers’ is 100 units, and the height of the tower above the horizontal is 40 units.
* Systematically explore the length of the cables required for different combinations of where the cables are attached

**Context 3: Pythagoras with other shapes** * Go to slide 12, which shows a diagram illustrating a right-angled triangle flanked with squares on all sides. Explain that by investigating the shapes using measurement (length) and spatial properties (area) and Pythagoras’ theorem, students can solve the following investigative problem.
	+ *Show that the sum of the areas of the two smaller half-square isosceles triangles is equal to the area of the larger half-square isosceles triangle. This can be done numerically rounded correct to 2 decimal places.*
* Ensure to visually refer to the triangles created within the squares by the diagonal lines.
* Suggest to students to start with a {6, 8, 10} right-angled triangle to find the relevant areas. Then suggest that they should experiment with several other right-angled triangles with half-square isosceles triangles on each edge to show consistent and relative results.
* The aim is to have students realise that the general result is true since the three sides of the right-angled triangle {*a*, *b*, *c*} are the base and height of the half-square isosceles triangles and that it is equivalent to multiplying each term in the equation $a^{2}+b^{2}=c^{2}$ by a factor of $\frac{1}{2}$ as the area of the half-square isosceles triangle is half the area of the square on which it is based.

**Shape 2*** Show that a similar method of investigation using Pythagoras’ theorem can be applied to the diagram using a right-angled triangle surrounded with semi-circles, as shown with slide 13. A useful online resource is [Pythagoras theorem with semicircles.](https://www.geogebra.org/m/GME9auEu)
	+ *Show that the sum of the areas of the two smaller semi-circles is equal to the area of the larger semi-circle. This can be done numerically rounded correct to 2 decimal places.*
* Suggest to students to start with a {6, 8, 10} right-angled triangle to find the areas, then experiment with several other right-angled triangles to calculate other diagonal lengths using Pythagoras’ theorem, for example, 12 cm and 16 cm side lengths. Provide the area of a circle if needed. Area of a circle: $area= π ×radius^{2}$ and $area=\frac{1}{4}× π ×diameter^{2}$. For a semi-circle, in terms of its diameter, the area is $\frac{1}{8}× π ×diameter^{2}$.
* The aim is for students to find that the general result is true since the three sides of the right-angled triangle {*a*, *b*, *c*} are the diameters of the semi-circles, and for these semi-circles the theorem is equivalent to multiplying each term in the equation $a^{2}+b^{2}=c^{2}$ by a factor of $\frac{π}{8}$.
* For students who investigate both shapes the takeaway learning is that if all terms on both sides of an equation are multiplied by the same positive factor, the resultant equation is equivalent.

Further notes for all investigations * A useful resource for students to visually see the effect of changing side lengths and hypotenuse is the [Basic Pythagoras](https://www.geogebra.org/m/UhWvHN3e#material/fhnrZGwX) interactive.
* To-scale diagrams are not required for the calculations, they may be useful to support some students. The use of 1 cm grid paper or equivalent digital application will be helpful to support construction of scale diagrams.
* Remind students that when they communicate their findings back to the teacher and class, that they are required to explain (briefly) the process they used to investigate their corresponding problems as well as what their finding revealed.
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| Summary and reflection15 mins | Invite groups to present a short 3-minute summary of their investigation, one for each context, followed by a couple of minutes discussion. This will need to have been coordinated with the groups as part of the lesson.Alternatively, the teacher could summarise findings based on observation of the student’s investigation. |
| Assessment | The following assessment opportunities are suggested below. * Display the Exit ticket (slide 14) and solutions (slide 15) from the teacher’s slides.
* Note: The key aspect of the exit ticket is the ability to read the information and construct a suitable right-angled triangle diagram. Students can either solve the problem using written working and a scientific calculator, or by using an online [Pythagoras calculator](https://www.geogebra.org/m/cxg3nk2m). In this context a suitable level of accuracy is one decimal place.
* Assess each group on their presentation skills and their ability to logically explain the mathematical approach taken to investigate their respective investigation.
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Further information

Provide any source materials for copyright purposes

Any further supplementary material, tables or drawings/images can be placed under here or in a separate document.

**Investigation context information**

**A selection of routine application questions**
[ALGEBRA (nuffieldfoundation.org)](https://www.nuffieldfoundation.org/sites/default/files/files/FSMA%20Pythagoras%20theorem%20student.pdf) - this resource provides some straightforward modelling problems which could be used to scaffold student familiarity with related contexts.