# Angles and parallel lines

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| Year levelStrand(s)Lesson lengthCD Code | * Year 7
* Measurement
* 50 mins
* [AC9M7M04](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8/content-description?subject-identifier=MATMATY7&content-description-code=AC9M7M04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)
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| Lesson summary | In this lesson, students will engage in various activities to explore angles between parallel lines in a navigational context. They will measure angles using traditional hand and finger techniques, construct parallel and perpendicular lines and establish laws of angle properties using algebraic formulas. Refer to the lesson [Stellar navigation and mathematics](https://www.ngarrngga.org/curriculum/stellar-navigation-and-mathematics)resource from Ngarrngga. The project aims to empower all teachers to integrate Aboriginal and Torres Strait Islander histories and cultures in their teaching. Find more resources at <https://www.ngarrngga.org/>. |
| Learning intention | * We can extend our knowledge of angle properties to identify corresponding, alternate and co-interior angle relationships.
* We can identify angles on parallel lines that are crossed by a transversal.
* We will apply new and existing knowledge and use critical thinking to solve measurement and spatial problems that exist in the real world.
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| Success criteria | By the end of this lesson students:have learnt and extended knowledge of angle relationshipscan apply knowledge of angle relationships between parallel lines to solve problems can describe the relationship using algebraic formulas can use critical thinking skills to solve problems in the real world  |
| Why are we learning about this? | We are learning about angles between parallel lines in a navigational context so we can precisely determine our orientation and location in space. While modern techniques like GPS have made navigation easier, exploring historical techniques enriches our understanding of geometry and how different tools contribute to exploring and understanding the world.  |
| Prerequisite student knowledge and language | Demonstrates that the angle sum of a triangle is 180˚ and can use this to solve problems.Identifies that angles at a point add to 360° and that vertically opposite angles are equal.Identifies interior angles in shapes to calculate angle sum.Uses angle properties to identify and calculate unknown angles in familiar two-dimensional shapes.Bisects/transverses a line or angle to cut it in half.The meaning of the terms perpendicular, bisect, intersection, corresponding angles, transversal and notation, are found on the [Version 9 mathematics glossary](https://v9.australiancurriculum.edu.au/downloads/learning-areas#accordion-b71b085f07-item-f1428171c5).  |
| Resources | Teaching slidesStar map worksheet (Word)Geometric activity (Word, print A3)Geometric assessment (Word)Provide coloured markers/pencils, rulers, compasses, protractors |

Curriculum information

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| Achievement standard | Students can apply knowledge of angle relationships between parallel lines to solve problems. Students can describe the relationship between angles using algebraic formulas. |
| Content description(s) | Identify corresponding, alternate and co-interior relationships between angles formed when parallel lines are crossed by a transversal; use them to solve problems and explain reasons. [AC9M7M04](https://www.figma.com/proto/VH6mibF9odkxlrOmEHK95p/Maths-Hub?type=design&node-id=100-1056&t=dzsRxDJGTcfg64OO-1&scaling=min-zoom&page-id=103%3A551&starting-point-node-id=100%3A1056) |
| General capabilitiesCross-curriculum priority | Numeracy Progression Understanding geometric properties ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8/general-capability-snapshot?subject-identifier=MATMATY7&content-description-code=AC9M7M04&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))identifies that angles at a point add to 360° and that vertically opposite angles are equal and reasons to solve problemsuses angle properties to identify perpendicular and parallel lines (for example, develops a computer-aided design drawing involving the creation of parallel and perpendicular lines)Critical and Creative Thinking Interpret concepts and problems ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8/general-capability-snapshot?subject-identifier=MATMATY7&content-description-code=AC9M7M04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))Draw conclusions and provide reasons ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7/general-capability-snapshot?subject-identifier=MATMATY7&content-description-code=AC9M7M04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=1&sub-element-code=CCTANAB&load-extra-subject=MATMATY7&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=60de4003-0af5-4ce4-a89e-2ed2bfe55fb2&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))Cross-curriculum priorityAboriginal and Torres Strait Islander Histories and Cultures ([A\_TSICP1](https://v9.australiancurriculum.edu.au/f-10-curriculum/cross-curriculum-priorities/aboriginal-and-torres-strait-islander-histories-and-cultures/slideout?code=A_TSICP1&organising-idea=0))First Nations communities of Australia maintain a deep connection to, and responsibility for, Country/Place and have holistic values and belief systems that are connected to the land, sea, sky and waterways |
| Areas of challenge | Some students may:mistakenly believe that the length of the lines that form an angle determines the size of the angle itself. For example, they might think that a larger angle is created by longer lines.experience difficulty in using a protractor accurately incorrectly starting their measure from 180° instead of 0°, and not lining up the rays of the angle correctly. Direct instruction and observation will help correct this misunderstanding and support the above misunderstanding.have learned about angles by rote learning. This can hinder students' ability to link geometric concepts. When students memorise properties without understanding the relationships between them, they may struggle to apply their knowledge flexibly. Focus on understanding rather than memorisation of these properties.  |
| Teaching strategies | [Mathematics investigation](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/mathematics-investigation/)[CRA](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/concrete-representational-abstract-cra/)[Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/) [Culturally responsive pedagogies](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/culturally-responsive-pedagogies/)[Setting goals](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/setting-goals/)  |

Lesson structure

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| Hook10 mins | Note: This class is based in the context of First Nations peoples who used hand and finger gestures to estimate distances between two objects and angle distances between stars in the sky and environment to orientate and navigate on land. To familiarise yourself with this concept refer to the [Stellar navigation and mathematics](https://www.ngarrngga.org/curriculum/stellar-navigation-and-mathematics) and scroll down to activity 1: measuring angles with your hands and fingers. Use the teaching slides to support this lesson.**Hook*** Explain to the class that hands and fingers have been used by Aboriginal and Torres Strait Islander people for millenia to measure angles between objects in degrees to help them navigate by following the stars at night. ‘Travel routes that can extend hundreds of kilometres were committed to memory through star maps and Songlines, one of the most famous being the Seven Sisters Songline, which stretches across the central half of the country,’ (D Hamacher, Allen & Unwin 2022 The First Astronomers, How Indigenous Elders read the stars, p.211) (slide 2).
* Explain that instead of rulers and protractors to make angle measurements, they used hand and finger gestures (like so) between two objects. Each gesture is a different angular distance measured in degrees and the distinctions between the gestures allowed First Nation peoples to make estimations on angles and distances to help them navigate their country and land.
* Show students the simplified hand symbols used by Aboriginal and Torres Strait Islander people to measure angles). Encourage students to follow along.
* Pose the thinking question on slide 3: if you measure the angle between the same two objects from a distance further away, will the angles be the same?
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| Explore15 mins15 mins | **Star map activities**Give out the Star map worksheet, one per student. Inform students of the learning intention of the lesson and introduce the first the activity: constructing a perpendicular bisect (slide 4). Introduce the Southern Cross and recognition by the International Astronomical Union (IAU) to name Ginan and other stars officially with their ancient cultural names (slide 5). This can be done by reading the introduction on the student instructions worksheet aloud. For a more interactive display, use the [Stellarium website](https://www.scootle.edu.au/ec/viewMetadata.action?id=A003050&q=&topic=&start=5180&sort=popularity&contentsource=&userlevel=&learningarea=&contenttype=&contentprovider=&resourcetype=&acVersion=&v=text&facetText=&facetsearchname=&showBookmarkedResources=&showLomCommercialResources=false&field=tags&commResContentType=all&commResContentType=%22App%20(mobile)%22&commResContentType=%22Audio%22&commResContentType=%22Book%20(electronic)%22&commResContentType=%22Book%20(printed)%22&commResContentType=%22Digital%20item%22&commResContentType=%22Learning%20object%22&commResContentType=%22Other%22&commResContentType=%22Printed%20item%22&commResContentType=%22Software%22&commResContentType=%22Teacher%20resource%22&commResContentType=%22Video%22&kc=any&lom=true&scot=true&follow=true&topiccounts=true&rows=20&suggestedResources=&fromSearch=true) to locate the Southern Cross and Southern Pointers (use the search bar to find it). Students can also access this website on their devices. The terminology of perpendicular, bisect, intersection, corresponding angles, transversal and notation is introduced to students as they complete their constructions. Writes terminology on whiteboard (optional slide 6). Students follow instructions to construct a perpendicular bisect/transverse line on their star map to find the South Celestial Pole (slides 7–8). Students then follow instructions to construct parallel lines on their star map (slide 9). Close activity by collecting the star maps for feedback. Have students take notes of the shared glossary chart and reiterate the new terminology that was learned during the activity. **Geometric reasoning exploration**Display slide 10 and place students in groups of three or four with the Geometric activity photocopied on A3 paper. Provide coloured markers but no protractors at this point. Students are given a particular shape with many angles where three angles are marked, and the others are not. Students use geometric reasoning to find any other angle that can be calculated given the three known angles – the discussion and thinking is more important than the answers. Initially get students to think individually then compare ideas with peers afterwards. It is important that students interact and support each other at tables. Students mark their angles with their markers. Stress that the only thing we can assume is that we know the size of the three indicated angles. Once the students have exhausted the possible angles they can find, provide additional information about pairs of lines that are parallel to prompt further discussion and establish parallel line angle relationships. Students describe the relationships they used (to differentiate/scaffold task students can use the colour of the angle in their description). Students decide on a variable that represents each colour and rewrites their angle relationships using algebraic formulas (slide 11). Revise how to use a protractor so students can check their solutions (slide 12). |
| Summary and reflection10 mins | **Consolidation task: angle relationships and terminology** Angle Relationships (slides 13–15), including an interactive demonstration of parallel line relationships [Angles in Parallel Lines – GeoGebra](https://www.geogebra.org/m/vuQ82CFW), refer to slide 6 for glossary and definitions.‘Notes for’ future self can be created by students on angle relationships established and terminology from the glossary terms. This could be done in a graphic organiser or in their exercise books. |
| Assessment | Star map activities: submitted for student portfolios. Download and handout the Geometric assessment and spend a few minutes explaining what students should do. This can be completed in the remaining time in your lesson or completed for homework. Slide 16 shows an example of a student worked example for one identified reason and argument. Further student examples are available for download. Students submit answers to this task.**Differentiation** (enabling and developing): note the assessment can be modified to suit groups or individual students.**Differentiation** (extending): [Angles Inside website](https://nrich.maths.org/13644) |

