Lesson plan: How wrong?

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| Year levelStrand(s)Lesson length CD Code | Year 10Measurement60 mins[AC9M10M04](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/content-description?subject-identifier=MATMATY10&content-description-code=AC9M10M04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| Lesson summary | In this lesson, students explore the importance of measurement errors by considering the ‘coastline paradox’ and real-world applications, including in aviation, sports and space exploration. They learn how measurement errors compound in volume calculations by exploring the impact of rounding when finding the volumes of prisms and cylinders. |
| Learning intention | * We are learning to develop a deeper understanding of the significance of measurement accuracy.
* We are learning about the compounding effects of rounding errors in real-world applications.
* We will use critical thinking skills in problem-solving.
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| Success criteria | By the end of this lesson, students can:give real-world examples where measurement error is problematic and hence why measurement accuracy is importantdemonstrate proficiency in rounding measurements to a specified degree of accuracy, calculating volumes pf prisms and cylinders and quantifying measurement errors in absolute and percentage termsevaluate the impact of measurement errors on volume calculations by explaining how rounding affects the accuracy of resultsengage in critical thinking and problem-solving by actively participating in discussions, asking relevant questions, and effectively communicating their findings and insights.  |
| Why are we learning about this? | One example where measurement is important is in planning a journey, whether by finding the time taken by public transport or in determining the speed of our car and whether we will have enough fuel. Small errors at different stages in our plans could mean we arrive late or not at all. As this lesson shows, there are many other practical scenarios where measurement errors can (and have had) significant or even disastrous consequences.  |
| Prerequisite student knowledge and language | It is expected that students can:find the volume of a prism or cylinder, using the formulae: $V=lwh, V=Ah$ and $V=πr^{2}h$round numbers to one or two decimal places and to the nearest whole numbercalculate one amount as a percentage of another amount. |
| **Resources** | Teacher’s slides (PowerPoint)Excel calculator (Excel)Worksheet 1 (Word)Worksheet 2 (Word) Exit ticket (Word)Post-it notes (one per student), access to computers |

Curriculum information

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| Achievement standard | Students identify the impact of measurement errors on the accuracy of results. |
| Content description(s) | Students identify the impact of measurement errors on the accuracy of results in practical contexts. ([AC9M10M04](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/content-description?subject-identifier=MATMATY10&content-description-code=AC9M10M04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) |
| General capabilitiesCross-curriculum priority | Numeracy:* Understanding units of measurement ([Level 10](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M04&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUuM&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Multiplicative strategies ([Level 10](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M04&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNMuS&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Critical and Creative Thinking:Interpret concepts and problems ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))Identify, process and evaluate information ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M04&general-capability-code=CCT&element-code=CCTINQ&sub-element-index=0&sub-element-code=CCTINQB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) Reflecting: Transfer knowledge ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10/general-capability-snapshot?subject-identifier=MATMATY10&content-description-code=AC9M10M04&general-capability-code=CCT&element-code=CCTREF&sub-element-index=0&sub-element-code=CCTREFB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) |
| Areas of challenge | Some students believe that all measurements are accurate and exact – obtain measuring instruments with different accuracy (for example, rulers with marks at 1cm, 0.5cm and 1mm) and use them to measure the same item, explaining that each time the measurement gets more accurate. Ask if the measurements would be more accurate if we could use a tool measuring to the nearest 0.5mm, 0.1mm and so on.Students may have difficulty finding the volumes of the various 3D solids and in recalling the relevant formulae – support students by using additional examples and providing practice in finding volumes in advance. Display formulae in the room, along with the names and diagrams (or reference sheets) of the relevant solids.Some students may struggle to access the complex calculations involved in finding the percentage error with several different degrees of accuracy – use the scaffolded version of the You try question on Worksheet 1; or simplify the number of calculations, for example, by rounding numbers to only one decimal place.Students may inadvertently use the diameter of the cylinder in volume calculations – support students with teacher reminders, the use of questioning such as ‘What is the $r$ in the formula?’, and having students work in pairs. |
| Strategies | [Worked examples](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/worked-examples/)[Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/) |

Lesson structure

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| Learning hook10 mins | Download the teacher’s slides found in the ‘What you need’ section. Coastline paradox(Slide 2) Distribute post-it notes and ask students to write their best estimate in kilometres to answer the question: ‘How long is the coastline of Australia?’Draw a rough number line on the whiteboard. Mark around five equal intervals, labelling the left-hand end ‘0’ and marking up to ‘50,000’. Students can use the scale on the number line to place their estimate in intervals of 10,000. Ask students what they notice. Identify the most common estimate and the range. Refer students to these measurements and references to compare to their estimate:* + 19,320km, The Australia Handbook
	+ 25,760km Wikipedia
	+ 34,000km (including islands), Geoscience Australia
	+ 30,270km or 47,070km (including islands), CSIRO Division of Land Use Research in Canberra.

Ask: *Why do you think there are so many different answers?* Answers may include different definitions (including or excluding islands, distance measured inland for estuaries), changes in coastline owing to erosion or land reclamation, better measuring tools and technologies. All of these are valid explanations.Use slide 3 to explain that this issue is known as the ‘coastline paradox’. Explain that one of the biggest factors that contributes to the variance is the scale of the measuring instrument (visualise this as the size of the ruler) you use to measure the coast. A smaller ‘ruler’ is better able to get into the nooks and crannies of the coastline and so will record a longer measurement than a longer ‘ruler’. This is because the coastline has an irregular curved shape, similar to a mathematical concept known as a fractal curve. Key point: How we measure things matters. The tools we use, the units we use and the degree of accuracy or rounding we employ can give rise to different answers. Emphasise that any measurement is necessarily rounded, as measured variables such as length, mass and time are examples of continuous data. |
| Explore40 mins | Ask students: *Why does measurement accuracy matter?* *Looking at the images, can you explain how inaccuracies in measurement could be an issue?* Use the examples to discuss errors in measurement.(Slide 4)Use worked examples to investigate the effect of compounding measurement errors in volume calculations.Explain that we are going to explore how rounding in measurements can have a compounding impact once rounded numbers are included into calculations, especially involving multiplication. Volume is important when measuring, for example, the capacity of an aeroplane’s fuel tank. We’ll use volume as our example, as calculating volume typically involves multiplying together three sets of measurements (as it relates to a three-dimensional shape).Distribute Worksheet 1 and show slides 5 and 6. Give students a few minutes to read and absorb the calculations for finding the volume of the following rectangular prism using the given measurements, rounding to two decimal places, one decimal place and the nearest whole number. They then determine the percentage errors generated using the rounded figures.Walk through the calculations using questions to determine and consolidate student understanding such as: ‘What did I do at this step?’ ‘Where has this number come from?’ ‘Why am I subtracting 1.76?’ ‘Why did I divide by 1.81504378?’ Students may need a refresher and practice in finding the volume of a rectangular prism, rounding, or expressing an amount as a percentage of another amount if they are struggling to understand aspects of the worked example.Display slide 7, which shows a different rectangular prism. Ask students to vote on what they think is the most likely answer to the question: ‘Will we get the biggest error when we round to two decimal places, one decimal place or to the nearest whole number?’Instruct students in pairs to repeat the process shown in the worked example to answer the question and to notice how Worksheet 1 breaks the task down into the required steps.**Differentiation** (support): There is also a heavily scaffolded version of the question for students who might initially struggle with the task.Check students’ answers using slide 8 and explore the answer to question 4. A typical student response might be: ‘Rounding to the nearest whole number gives a much bigger error and % error because the numbers change quite significantly, for example, $4.574 ≈5$. You would expect this would always be the case as the more you round a number the less accurate your rounded value becomes.’**Differentiation** (extension): Display slide 9: An unexpected result. Ask different groups of students to calculate the volume using the given values – values rounded to two decimal places, one decimal place and to the nearest whole number. Have a student from each group present their calculations on the board and calculate the various errors or display the table on the slide that shows the different volumes and errors. The unexpected result is that the values rounded to the nearest whole number give a more accurate estimate of the volume (1.04% error) versus those rounded to one decimal place (4.41% error). * + This reflects the impact of compensating errors: the length and width are both close to 3.95m and 3.05m and when rounded to one decimal place they both are rounded up. However, when rounded to the nearest whole number, the first rounds up to 4m and the second down to 3m so the differences compensate to reduce the net error when calculating the volume.
	+ This is a relatively rare phenomenon and, in most cases, the greater the degree of accuracy in your measurements, the closer to the actual volume your calculation will be.
	+ Students could use a spreadsheet such as the Excel calculator to experiment with finding different dimensions that give similar results.

**Paired work**Display slide 10 and explain that there are various different 3D solids with measurements given to two or three decimal places. Students choose at least three different solids and find the volumes. They then repeat this process using rounded values and calculate the error and percentage error.* Ask students to name the different solids and explain how they would find the volume. In general, the volume of a prism is found using the formula $V=Ah $where *A* is the cross-sectional area and $h$ is the height of the prism. Refer to slide 10 notes for relevant formulae.
* Distribute Worksheet 2 and monitor students as they proceed with the activity.

**Differentiation** (support): student choice allows for students to choose more accessible shapes such as the cube and rectangular prisms or more challenging shapes such as the trapezoidal prism or annular cylinder. Students can also choose whether they explore using only one set of rounded dimensions (or more) for their chosen solid. |
| Summary and reflection10 mins | Review answers using the Excel calculator: Worksheet solutions tab. Show students slide 11 and ask them to write down three things they learned today. Monitor students’ responses and select three students to contribute their ideas to be recorded on the board. Key points you may want to highlight include:* Measurement errors arise because all measurements are rounded (continuous data).
* More accurate instruments reduce measurement error (the coastline paradox).
* There are many practical situations where accurate measurement is important.
* The more you round a number (that is, to fewer decimal places) the less accurate it becomes.
* In volume calculations, measurement errors compound as rounded numbers are multiplied together.
* Errors can become quite significant (e.g. 10% or even 20%) when small numbers are rounded to the nearest whole number and then multiplied.
* When calculating a % error you divide by the original (most accurate) volume.
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| Assessment | An assessment task is provided in the exit ticket. * Distribute the Exit ticket for students to complete.
* Note that there are identical three exit tickets per A4 page.

Atif says, ‘Rounding doesn’t matter when working with measurements as the rounded values are close enough.’Create an example to help explain your ideas.Here's an example a student may use to illustrate Atif's statement:Let's say we have a measurement of 12.345 meters. If we round this measurement to the nearest whole number, we get 12 meters. Atif's statement implies that using 12 meters instead of 12.345 meters is close enough for most practical purposes, such as estimating materials needed for a project or measuring distances in everyday situations. |