YuMi Deadly Maths

Year 8 Teacher Resource:

SP – One word changes it all

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

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Year 8 Statistics and Probability

One word changes it all

Learning goal Students will:

- describe events using ‘and’, ‘or’ and ‘not’ language
- develop and use the probability formulas for simple ‘and’, ‘or’ and ‘not’ events.

Content description Statistics and Probability – Chance

- Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’ (ACMSP205)

Big idea Probability – Probability as fraction

Resources Coin, dice, cards for coin faces and two sets digits (1 to 6), Maths Mat, elastics, watches, packs of cards

Reality

Local knowledge Discuss situations in the local environment where two situations may both occur, where one or the other may occur or where one may occur but not the other; e.g. Tomorrow may be sunny and windy, sunny or raining, sunny but not hot.

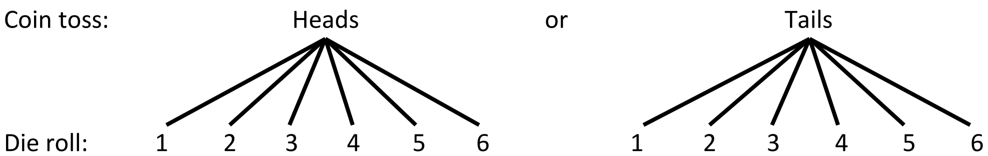
Prior experience Check students’ understanding of probability terms. See **Appendix A** for definitions of some terms. Ask students:

- *Why do we conduct chance experiments?* [To help us calculate the chance of an event occurring in the future.]
- *How is probability calculated?* [By dividing the number of favourable outcomes by the total number of possible outcomes.]
- *How is this written algebraically?*

  Probability (of an event) = $\frac{\text{(number of favourable outcomes)}}{\text{(total number of possible outcomes)}}$

Kinaesthetic Discuss the probability of tossing a coin and rolling a die.

Have two students holding appropriate cards represent the faces of the coin and 12 students with numeral cards represent the roll of the die (6 for heads and 6 for tails).



Pose questions such as:

- *What is the total number of outcomes in this sample space?* [12]
- *How do you know this?* [There are two possible outcomes for the coin toss and six possible outcomes for the die roll and 2 × 6 = 12, or simple addition: H1 + H2 ... + T5 + T6 = 12.] Draw a two-way table on the board to show all the possible outcomes.
- *What is the probability of tossing heads **and** rolling a four?* Student with the 4 card under the Heads raises the card. On the two-way table, circle H4 as shown:

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

[P(H and 4) is 1 favourable outcome out of 12 total = $\frac{1}{12}$. This is because the outcome has to be **both** heads **and** 4 which is one card only. Note that this is the same as P(H) × P(4) = $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$]

- *What is the probability of tossing tails **and** rolling an odd number?* Students with odd numbers under Tails raise their cards: T1, T3, T5. Use the two-way table:

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

[P(T and odd) = 3 favourable outcomes out of 12 total = $\frac{3}{12} = \frac{1}{4}$. This is because the outcome has to be **both** tails **and** the three odd numbers which gives three outcomes. Note that this is the same as P(T) × P(odd) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$]

- *What is the probability of tossing heads **or** rolling a 5 **or** both?* Students with Heads cards raise them and also the student with the 5 card in Tails. Use the two-way table:

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

[P(H or 5 or both) = 7 favourable outcomes out of 12 total = $\frac{7}{12}$. This is because the outcome can be **either** heads **or** a 5 **or** both so there are 6 heads {one of which is a 5} and a 5 in the tails, 6 + 1 = 7 favourable outcomes. Note that this is the same as P(H) + P(5) – P(H and 5) = $\frac{6}{12} + \frac{2}{12} - \frac{1}{12} = \frac{7}{12}$ as all possible outcomes should be counted once only.]

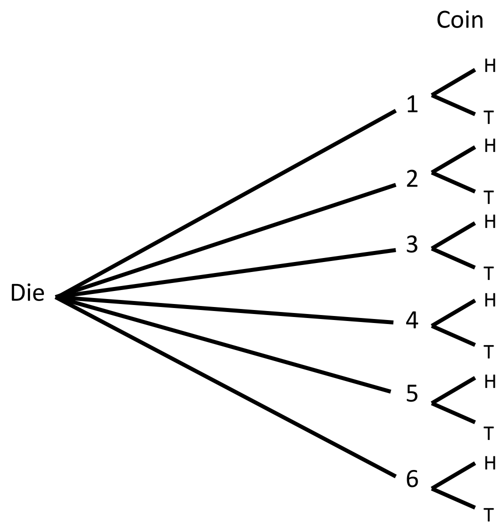
- *What is the probability of tossing heads and **not** rolling a 2 or 3?* Students with Heads cards except the students with 2 and 3 raise their cards. Use the two-way table:

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

[P(H and not 2 or 3) = 4 favourable outcomes out of 12 total outcomes = $\frac{1}{3}$. This is because all the heads are being considered except/**not** the 2 and 3 of the heads, which leaves the other four → H1, H4, H5, H6. Note that this is the same as P(H) × P(not 2 or 3) = $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$]

Give similar examples so that students gain the understanding of the impact of 'and', 'or', and 'not' in probability. One word makes all the difference or changes it all.

Using the students, construct a tree diagram to demonstrate the possible outcomes (the diagram on right uses the same sample space but with the die roll first and then the coin toss).

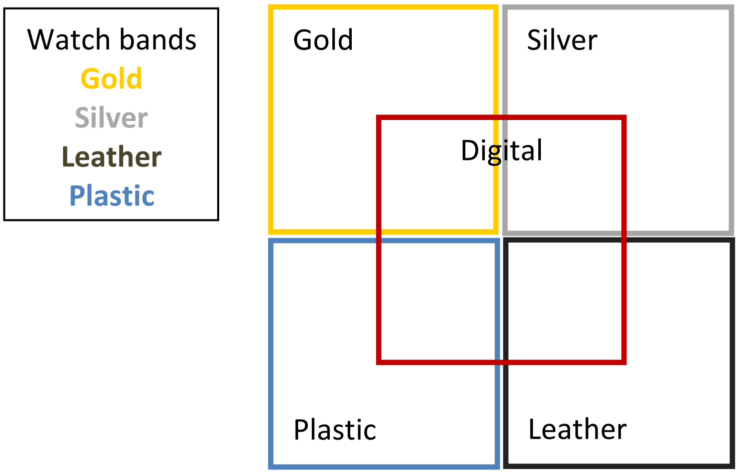


Abstraction

Body

Check to find the students who have gold, silver, leather, plastic bands on their watches (or use different types of shirts – buttons, polo, t-shirts, shirts with a tie; or shoes – laces, buckles, Velcro, slip-on). Divide the Maths Mat into four sections with the appropriate watch band colours. Have students stand in one of the four sections according to whether their watch band is gold, silver, leather or plastic. Construct a square with a red elastic at the centre of the mat so that it intersects all quadrants. Students with digital watches move into the intersection leaving students with non-digital (analogue) watches outside the intersection.

Watches worn by Year 8 students in this class



Have the students who don't wear watches construct a two-way table, inserting relevant numbers tallied from each section of the Maths Mat.

Watches worn by Year 8 students in this class

Bands		Non-digital	Digital
	Gold		
	Silver		
	Leather		
	Plastic		

Pose questions such as:

- *What is the total number of outcomes for all watch types?*
- *What is the probability of students wearing a digital watch with a metal (gold or silver) band?*
- *What is the probability of students wearing a non-digital watch?*
- *What is the probability of students wearing a leather watch band?*
- *What is the probability of students wearing a digital watch with a silver or plastic band?*

Reverse:

- *Which type of watch has a probability of x?* (x = fraction determined by number of students in the selected type)
- Extension: *What is the probability of students who do not wear a watch?* (Note that the total number of outcomes has changed to the total number of students in the class.)

Hand Packs of cards: 52 cards in a pack, 4 equal suits with 2 black suits (spades and clubs) and 2 red suits (hearts and diamonds). Working with partners, students create two-way tables (see examples below) to help answer the following (or similar) probability questions:

What is the probability of drawing:

- *a black card?*
- *a diamond?*
- *not a diamond?*
- *a red four?*
- *the jack of clubs?*
- *a seven?*
- *a spade or a six or both?*
- *a spade or a six but not both?*
- *a ten that is not a diamond?*
- *a green card?*

(See **Appendix B** for answers to the above questions.)

Two-way table for suits and colours:

		Colour	
		Red	Black
Suit	Spades		13
	Clubs		13
	Hearts	13	
	Diamonds	13	

Two-way table for suits/colours and numbers:

		Card												
		1	2	3	4	5	6	7	8	9	10	J	Q	K
Suit	Spades	1	1	1	1	1	1	1	1	1	1	1	1	1
	Clubs	1	1	1	1	1	1	1	1	1	1	1	1	1
	Hearts	1	1	1	1	1	1	1	1	1	1	1	1	1
	Diamonds	1	1	1	1	1	1	1	1	1	1	1	1	1

Mind *In your mind, see the probability of drawing a hearts card from the pack. Now see the probability of drawing the queen of hearts (queen* ***and*** *hearts), then the probability of drawing a queen* ***or*** *hearts (or both) from the pack, and finally the probability of drawing a queen but* ***not*** *hearts. What is a possible question about drawing cards from a pack that would give a probability of:* $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{13}$, $\frac{1}{52}$, $\frac{4}{13}$?

Creativity Students describe a two-step experiment, pose questions and calculate probabilities in a two-way table.

Mathematics

Language/ symbols	probability, experiment, trial, event, and, or, not, favourable, unfavourable, outcome, sample space

Practice

1. Following from the activities in Reality and Abstraction, develop the probability formulas for calculating 'and', 'or' (both inclusive 'or' and exclusive 'or') and 'not' probability events:

   P(A and B) = P(A) × P(B)

   P(A or B or both) = P(A) + P(B) – P(A and B)

   P(A or B but not both) = P(A) + P(B) – 2(P[A and B])

   P(not A) = 1 – P(A)

2. Give students exercises/worksheets that require calculation of probabilities using the above formulas.

3. Play the following probability games.

   A. Feud

   Materials: two dice, two players.

   Rules:

   (a) Players in turn throw dice and add the numbers.

   (b) If the point sum is 2, 3, 4, 10, 11 or 12, player 1 receives one point. If the sum is 5, 6, 7, 8 or 9, player 2 receives one point.

   (c) The first player to 10 points wins.

   Questions (after many games):

   (a) *Is the game fair?*

   (b) *Does it matter whether you are player 1 or 2?*

   (c) *What extra numbers could we give player 1 to even the contest?*

   B. Two-dice difference

   Materials: Two dice, pad and pencil.

   Experiment: Roll two dice. Calculate the difference between the uppermost faces – take low from high.

   Questions: *What difference is likely to occur most frequently? Least frequently? What differences can occur? What is the probability of these differences?*

   Procedure:

   (a) Investigate these questions and record your results and findings on pad.

   (b) Devise an experiment to test your conclusions. *What did you find? Were your expectations confirmed? How could you improve your experimental procedure?*

   (c) *If you were the banker in a gambling game of two-dice difference, what odds would you strike for each difference?*

Connections Connect probability to fractions, decimals, ratio, percentage.

Reflection

Validation

Students discuss where probability is used in the real world and validate their partner's two-step event from Creativity above.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently.

Extension

Flexibility. Students use tree diagrams and two-way tables to represent two-step events in order to calculate probabilities of:

- one event **and** another event occurring;
- one event **or** another event **or both** occurring;
- one event **or** another event **but not both** occurring;
- an event **not** occurring; and
- one event but **not** another event occurring.

Reversing. Students are able to move between events ↔ tree diagram ↔ two-way table ↔ probability, starting at any given point.

Generalising. Probability is always calculated by dividing the number of favourable outcomes by the total number of possible outcomes. The use of 'and' means the favourable outcomes must satisfy both criteria and multiplication is used to calculate the probability; the use of 'or' gives an addition of the probabilities of both criteria while ensuring that all possible outcomes are counted once only; the use of 'not' means those stated must be excluded or subtracted in calculating favourable outcomes to calculate probability. Tree diagrams and two-way tables are useful tools for showing all possible outcomes and the intersection of the events.

Changing parameters. Increase the complexity of the contexts by extending to three-step events, e.g. 'and, and'; 'and, or'; 'and, but not'; 'or, but not'. Play the Planetfall game then discuss the question at the end:

Planetfall

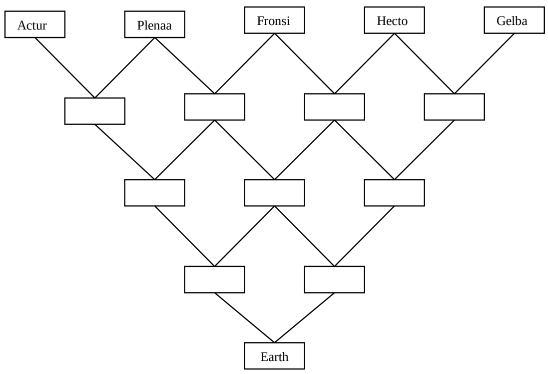
Materials: one coin, counters, board as below right, 2–6 players.

Rules:

(a) Players place counters (spaceships) at start (Earth).

(b) Players in turn toss the coin and move left if heads and right if tails.

(c) Players score one point for reaching Fronsi, two points for reaching Plenhaa or Hecto and three points for reaching Gelbt or Actur.



(d) The first player to make 10 points wins.

Question (after many games): *What is the most likely planet to reach? Why?*

Teacher's notes

- Ensure that students have a sound understanding of calculating the probability of single-step events before proceeding to the probability of two-step events.
- Check that students understand the language differences of 'and', 'or' and 'not' in describing favourable outcomes in events.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of an item, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different item.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

Appendices

Appendix A: Probability terminology

Experiment: A process involving chance that leads to results called *outcomes*. It can have one or more steps/stages and be repeated many times (*trials*). An example of a probability experiment would be tossing a coin three times and recording the outcomes in order, e.g. HTT, THT, etc. Note that this is different from an experiment to toss a coin three times and record, for example, the number of tails.

Trial: A single repeat of a probability experiment. An experiment can consist of one or more trials. For example, if an experiment of tossing a coin three times and recording the outcomes in order is repeated many times, each repeat is called a trial.

Outcome: The result of a single trial of a probability experiment. For example, a single coin toss experiment has two possible outcomes: heads or tails. An experiment of tossing a coin three times and recording the outcomes in order has eight possible outcomes (2 × 2 × 2): HHH, HHT, HTT, HTH, TTT, TTH, THH, THT.

Sample space: The set of all possible outcomes of the probability experiment and defined by the experiment. The sample space of an experiment of tossing a coin three times and recording the outcomes in order is the eight possible outcomes listed above.

Event: The selected outcome being investigated that is one or more of the possible outcomes in the sample space. In an experiment of tossing a coin three times and recording the outcomes in order, we might wish to investigate the probability of tossing the sequence HHH – this is the event being investigated.

Appendix B: Answers to pack of card questions in Abstraction – Hand

Question	**Answer**
Probability of drawing a black card	P(black) = $^{26}/_{52} = ^{1}/_{2}$
Probability of drawing a diamond	P(diamond) = $^{13}/_{52} = ^{1}/_{4}$
Probability of not drawing a diamond	P(not diamond) = $1 - ^{1}/_{4} = ^{3}/_{4}$
Probability of drawing a red four	P(red and 4) = $^{2}/_{52} = ^{1}/_{26}$ Note that this is the same as P(red) × P(4) = $^{1}/_{2} \times ^{1}/_{13}$
Probability of drawing the jack of clubs	P(J and clubs) = $^{1}/_{52}$ Note that this is the same as P(J) × P(clubs) = $^{1}/_{13} \times ^{1}/_{4}$
Probability of drawing a seven	P(7) = $^{4}/_{52} = ^{1}/_{13}$
Probability of drawing a spade or a six or both	P(spade or 6 or both) = $^{13}/_{52} + ^{4}/_{52} - ^{1}/_{52} = ^{16}/_{52} = ^{4}/_{13}$ Note that the probability of drawing the six of spades ($^{1}/_{52}$) is subtracted because it is included in both P(spade) and P(6) and all possible outcomes should be counted only once.
Probability of drawing a spade or a six but not both	P(spade or 6 but not both) = $^{13}/_{52} + ^{4}/_{52} - ^{1}/_{52} - ^{1}/_{52} = ^{15}/_{52}$ Note that the probability of drawing the six of spades ($^{1}/_{52}$) is subtracted twice because it is included in both P(spade) and P(6) and both need to be excluded.
Probability of drawing a ten that is not a diamond	P(10 but not diamond) = $^{4}/_{52} - ^{1}/_{52} = ^{3}/_{52}$ Note that this can also be calculated using multiplication in the same way as the 'and' probabilities: P(10 and not diamond) = $^{1}/_{13} \times ^{3}/_{4} = ^{3}/_{52}$
Probability of drawing a green card	P(green) = 0

