Making triangles

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| Year level  Strand(s)  Lesson length  CD code | 7 / 8  Space, Measurement  60 min  [AC9M8M01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8M01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick), [AC9M8SP01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8SP01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) , [AC9M8SP04](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=1&subjects-start-index=0&view=quick) |
| Lesson summary | Students study the concept of triangle inequality, which determines if three positive numbers can serve as the side lengths of a triangle. This principle states that a triangle is possible if the largest of the three numbers is smaller than the sum of the other two. To explore this, students experiment with various combinations of three natural numbers. They investigate whether these numbers can form a triangle and, when they do, students classify and construct the corresponding triangle. Additionally, they learn to calculate the perimeter of these triangles.  This lesson also reviews Year 7 content to enable students to successfully engage in four related lessons that explore the Year 8 Pythagoras Theorem content. |
| Learning intention | * We are learning to describe and explain the triangle inequality. * We will formulate a mathematical statement. * We will show this principle in action using examples and counterexamples. |
| Success criteria | By the end of this lesson, students can:   * classify a triangle as equilateral, isosceles or scalene from a diagram with given side lengths and calculate its perimeter * identify when three positive integers can be used as side lengths of a triangle and construct the corresponding triangle using geometry software or ruler and compass * state and explain the triangle inequality. |
| Why are we learning about this? | Triangles, the simplest plane shapes, have widespread applications in various fields such as art and design, cartography, surveying, navigation, architecture, construction, and facial recognition algorithms. To uniquely describe a triangle, one effective method is to specify the lengths of its three sides. This method is known as the Side-Side-Side (SSS) congruence condition, which states that if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent. |
| Prerequisite student knowledge and language | Prior to this lesson, it is expected that students have:  familiarity with the spatial terms point, line, angle, side and triangle, and their representation and labelling in diagrams  knowledge of the classification of triangles as equilateral, isosceles or scalene according to the number of sides of equal length  familiarity with the measurement terms degree, unit, cm, perimeter  familiarity with the number terms positive, integer, inequality. |
| Resources | Teacher’s slides (PowerPoint)  Student investigation worksheet (Word)  Straws or pipe cleaners, rulers, compasses, 1-cm-grid graph paper, laptops  Dynamic geometric software (optional) |

Curriculum information

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| Achievement standard | [Students classify polygons according to their features and create an algorithm designed to sort and classify shapes (Year 7).](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7?view=quick&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0)  [They identify conditions for congruency and similarity in shapes and create and test algorithms designed to test for congruency and similarity (Year 8).](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8?view=quick&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=1&subjects-start-index=0) |
| Content description(s) | Note: this lesson primarily is primarily for the Year 8 level however it also reviews related Year 7 content, listed in the first dot point below.   * Students classify triangles, quadrilaterals and other polygons according to their side and angle properties; identify and reason about relationships. [AC9M7SP02](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-7/content-description?subject-identifier=MATMATY7&content-description-code=AC9M7SP02&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) * They solve problems involving the area and perimeter of irregular and composite shapes using appropriate units. [AC9M8M01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8M01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) * Students identify the conditions for congruence and similarity of triangles and explain the conditions for other sets of common shapes to be congruent or similar, including those formed by transformations. [AC9M8SP01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8SP01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) * They design, create and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works. [AC9M8SP04](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| General capabilities  Cross-curriculum priority | **General capabilities**  Numeracy   * Properties of shapes and object ([Level 4](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-7/general-capability-snapshot?subject-identifier=MATMATY7&content-description-code=AC9M7SP02&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) * Understanding units of measurement ([Level 9](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M01&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUuM&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) ) * Understanding geometric properties ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP01&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) * Proportional thinking ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP01&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNPrT&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))   Critical and Creative Thinking   * Draw conclusions and provide reasons ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-7/general-capability-snapshot?subject-identifier=MATMATY7&content-description-code=AC9M7SP02&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) * Interpret problems and concepts ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) * Evaluate actions and outcomes ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=2&sub-element-code=CCTANAC&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) * Consider alternatives ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&general-capability-code=CCT&element-code=CCTGEN&sub-element-index=0&sub-element-code=CCTGENB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))   Digital literacy  Select and operate tools ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=DL&element-code=DLMO&sub-element-index=0&sub-element-code=DLMOC&elaboration-code=AC9M8N01_E2&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) |
| Areas of challenge | * Students might assume that any set of three positive numbers can form a triangle. To address this misconception, teachers can provide carefully crafted activities. These activities might include experimenting with different numeric combinations that both can and cannot form triangles, or reviewing a selection of pre-constructed examples and counterexamples. Such methods help enhance students' understanding of triangle formation. * Visualising triangles based on given side lengths can be challenging for students. To support their learning, hands-on activities with preconstructed line segments (concrete manipulatives) of various lengths can be effective. Students can experiment by combining sets of three segments to form triangles. Additionally, digital tools offering virtual manipulatives can further aid in visualising these shapes. * Recalling the definitions of different types of triangles might be difficult for students. A helpful solution could be a classroom poster featuring a classification flowchart with examples of each triangle type. Also, concrete models of triangles, such as those made from cardboard or plastic, can be effective in reinforcing the characteristics and differences of various triangle types. |
| Strategies | [Mathematics investigation](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/mathematics-investigation/)  [Questioning](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/questioning/)  [Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/)  [Concrete, Representational, Abstract (CRA)](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/concrete-representational-abstract-cra/) |

Lesson structure

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| Learning hook  20 mins | The introduction and exploration activities require students to be guided through the construction of a triangle given its three side lengths using a selection or combination of manipulatives and physical and digital drawing tools (for example, straws, pipe cleaners, flexible/bendy rods, ruler and compass, or geometry software.)  **Learning hook**  This activity is designed to stimulate interest and have students engage in sketching a cartoon face by and connecting triangular shapes.  Prompt: how can you use only triangles to sketch a face? Start with a rough freehand sketch on a whiteboard and/or show a simple pre-prepared example (slide 4).  Students have a go at using only triangles to sketch/draw a cartoon style face (showing features of eyes, mouth, nose, ears, hair) of their own, share these in small groups and discuss how they developed their cartoon face.  Some related questions/points for discussion are:  What did you start with?  What is the overall shape of the cartoon face?  How were certain facial features represented, for example nose, eyes, hair?  (Comment that this activity informally links to some of the considerations in the use of triangulation in facial recognition.)  **Triangles in the real world**   * Invite students to provide examples of triangles in the real world from their experience and knowledge and form a list of these.   Show several examples of practical applications of triangles (slides 5–9: facial recognition, bridges, buildings, art and design, surveying) commenting briefly on each one, and connect these to/supplement the list of students' examples as applicable.   * Observe that any three distinct points in the plane that (not all in the same straight line) form a triangle (slide 10). Comment that one can measure the side lengths and then calculate the perimeter of a triangle. * Demonstrate the construction of a triangle with side lengths {9, 14, 15} using either a ruler and compass or geometry software as applicable, and have students co-construct their own version. Have students calculate the perimeter of this triangle (9 + 14 + 15 = 38). [Triangle construction](https://www.geogebra.org/m/PC2fbDmr) or [Triangle maker](https://www.geogebra.org/m/JHgTXKrt) * Ask students to individually construct a triangle with side lengths {10, 15, 17} – using cm as the basic unit, and in small groups check visually that their triangle ‘looks the same’ as that of other students. Note that giving the three side lengths of a triangle uniquely specifies the triangle, this is the Side, Side, Side or SSS congruency condition. * Slide 11 shows a corresponding triangle diagram. A common practice example such as this can be used by the teacher to check that students can suitably construct a triangle. Have students calculate the perimeter of the triangle (10 + 15 + 17 = 42). * Review the classification of triangles as equilateral (3 equal length sides), isosceles (2 equal length sides) or scalene (0 equal length sides), this can be done by asking students if they recall the names of the triangle types. |
| Explore  30 mins | * Pose the question: If you are given three positive numbers, can they always be used as the side lengths of a triangle? * Distribute theInvestigation worksheet (slide 12) to have students, individually or in small groups, explore which given combinations work, construct the corresponding triangle for those that do, classify the triangle, and calculate its perimeter. The Worksheet includes the {10, 15, 17} example as its first row, and three additional rows for students to include their own {*a*, *b*, *c*} combinations. * Students work in small groups to discuss and describe the relationship between the lengths of the three sides *when* a triangle can be formed. * Ask each group to develop an example of a combination of three numbers that don’t form a triangle, for example, {8, 10, 20}, and use a diagram to show why this is the case (slide 13). Have students work in small groups to develop a written statement (slide 14) set of dot point conditions, for the triangle inequality, and how to apply it. These can be used to form the basis for an *algorithm.* * Note: for isosceles and equilateral triangles there will be sides of equal length, so the idea of ‘equal shortest’ length or ‘equal longest’ length could be used in these cases.   Further interactives: [Geometry - GeoGebra](https://www.geogebra.org/geometry?lang=en) (for general geometric constructions from first principles); [Triangle Inequality Exploration – GeoGebra](https://www.geogebra.org/m/JT5UmZ8w) |
| Summary and reflection  10 mins | Develop a simple set of conditions and a process or algorithm. (Slide 15shows provides an example.)  Step 1: Given three positive numbers order them from smallest to largest and label them {*a*, *b*, *c*} where *a* ≤ *b* *≤ c.*  Step 2: Calculate *a* + *b* and compare with *c.*  Step 3: If *a* + *b* > *c*, then a triangle can be formed, if *a* + *b* ≤ *c* then a triangle cannot be formed.  Use a diagram or digital manipulative to illustrate what happens when *a* + *b* ≤ *c* (a base with two ‘arms’ that don’t meet) and include the special case where *a* + *b* = *c* (the two arms form an identical segment to the base which lies exactly on top of it).  If time allows, use a digital tool to show three related examples with the same length for the longest side and one of the short sides in each ease.  {6, 8, 16} so *a* + *b* *<* c – no triangle can be formed.  {8, 8, 16} so *a* + *b* = c – no triangle can be formed.  {8, 10, 16} so *a* + *b* > c – a triangle can be formed.  (Iteration: Students create a classification flowchart poster. This poster can serve as a visual guide, starting with an initial check to determine if a set of three numbers can form a triangle. Following this, the flowchart can guide students to classify the type of triangle based on the number of equal-length sides: (0 = scalene, 2 = isosceles, 3 = equilateral). |
| Assessment | The following suggestion is given as an assessment opportunity.   * Download the teacher’s slides and go to slide 16, a table with several sets of 3 numbers. * Students answer ‘Yes’ or ‘No’ as to whether they can be used to form the sides of a triangle or not. * Students classify the type of triangle and calculate its perimeter when this is the case. |