Pythagoras: Cartesian coordinate plane

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| Year levelStrand(s)Lesson lengthCD Code | Year 8Measurement, Number60 mins[AC9M8M06,](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8M06&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) [AC9M8N01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8N01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick), [AC9M8SP04](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| Lesson summary | This lesson is one in a series of lessons on Pythagoras theorem. Students calculate the distance between any two points on the Cartesian coordinate plane using Pythagoras’ theorem and then develop a set of steps that outline the process. It combines the use of Pythagoras’ theorem with the Cartesian coordinate plane and the use of coordinates to specify position, form line segments, shapes of triangles and determine their perimeters. Suggestions are made throughout the lesson with respect to opportunities for exploration, questioning and reflection.  |
| Learning intention | * We are learning to draw triangles in the Cartesian coordinate plane using coordinates and joining them with line segments.
* We can find the distance of a point in the coordinate plane from the origin.
* We can find the length of a line segment joining two points in the coordinate.
* We will determine the perimeter of triangles in the coordinate plane.
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| Success criteria | By the end of this lesson, students can:* calculate the distance of a point from the origin in the coordinate plane
* find the length of a line segment in the coordinate plane
* calculate the perimeter of a triangle in the coordinate plane.
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| Why are we learning about this? | The Cartesian coordinate plane is one of the key mathematical representations of shapes in the plane, enabling us to specify location and construct shapes using coordinates. It connects numerical, graphical and algebraic information about position and shapes. These can be applied to solve problems involving position and distance in maps, surveying, design and construction. |
| **Prerequisite student knowledge and language** | Prior to this lesson, it is expected students have knowledge of the: * Cartesian coordinate plane, and the location of points using coordinates in all four quadrants
* spatial terms: point, coordinates, origin, axes, line segment, side, right-angle, quadrilateral, diagonal and their representation and labelling in diagrams
* measurement terms: distance, length, perimeter and area
* number terms and notations for integer, square and square root.
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| **Resources** | * Teacher’s slides (PowerPoint)
* A4 1-cm-grid graph paper (Word)
* Scientific calculator
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Curriculum information

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| Achievement standard | Students use Pythagoras’ theorem to solve measurement problems involving unknown lengths of right-angled triangles. Students recognise irrational numbers and terminating or recurring decimals. Students identify conditions for congruency and similarity in shapes and create and test algorithms designed to test for congruency and similarity.  |
| Content description(s) | Students recognise irrational numbers in applied contexts, including square roots and *π.*[AC9M8N01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8N01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)Students use Pythagoras’ theorem to solve problems involving the side lengths of right-angled triangles.[AC9M8M06](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8M06&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) They design, create and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works. [AC9M8SP04](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/content-description?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| General capabilitiesCross-curriculum priority | **General capabilities**Numeracy* Understanding geometric properties ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M06&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Understanding units of measurement ([Level 10](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M06&general-capability-code=N&element-code=NM&sub-element-index=1&sub-element-code=NMUuM&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Multiplicative strategies ([Level 9](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8M06&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNMuS&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Proportional thinking ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNPrT&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Critical and creative thinking* Interpret concepts and problems ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Consider alternatives ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-8_year-7_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=CCT&element-code=CCTGEN&sub-element-index=0&sub-element-code=CCTGENB&load-extra-subject=MATMATY7_MATMATY8_MATMATY9_MATMATY10&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&achievement-standard=04f8219e-c3b7-4ecf-a612-701f06ecf8f7&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Draw conclusions and provide reasons ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-7_year-8_year-9_year-10/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8SP04&general-capability-code=CCT&element-code=CCTANA&sub-element-index=1&sub-element-code=CCTANAB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Digital Literacy* Select and operate tools ([Level 5](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-8/general-capability-snapshot?subject-identifier=MATMATY8&content-description-code=AC9M8N01&general-capability-code=DL&element-code=DLMO&sub-element-index=0&sub-element-code=DLMOC&elaboration-code=AC9M8N01_E2&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
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| Areas of challenge | Some students may:* not yet understand the meaning of a coordinate, and order of coordinates in an ordered pair
* confuse how to represent points from their coordinates in each quadrant, interpreting the negative sign correctly; the resource: [Concept of Cartesian coordinates – GeoGebra](https://www.geogebra.org/m/sh54rzkt) can be used to help students gain fluency with interpreting coordinates and locating points on the Cartesian coordinate plane.
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| Strategies | [Questioning](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/questioning/)[Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/)[Concrete, Representational, Abstract (CRA)](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/concrete-representational-abstract-cra/)[Worked examples](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/worked-examples/)[Classroom talks](https://uat-www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/classroom-talks/) |

Lesson structure

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| Learning hook | Refer to the teacher’s slides to support the teaching of this lesson.**Learning hook**Show slide 4 and pose the statement to students: estimate the length of the diagonals of the square. Check that students know that word ‘Diagonal’ before they estimate. The square is 12 x 12 units. Give students some time to reflect on the question before having students share their estimates and approaches via a whole-class discussion.The expected student approaches are:estimation by eyecounting using a visual estimate of the number of unit segments that could fit inusing a multiple of the side length, for example, 1.5 times 12.Establish the actual length by either having yourself or a student working it out using Pythagoras’ theorem: $c^{2}= a^{2} + b^{2}$ to find length $c= \sqrt{(a^{2}+b^{2}})$. Note that use of the theorem provides an exact answer as square root, leading to a numerical approximation using a calculator.**Introduction**Display slide 5to launch the lesson’s focus and task: **Question:** How can we calculate the distance between any two points on the Cartesian plane? (across all four quadrants)**Task:** Develop a sequence of steps that outline the process for working out the distance between any given two points on the Cartesian plane.Go to slide 6. Distributethe A4 1-cm-grid graph paper downloadable from the ‘What you need section’ from the online lesson, and pose the following scenario: A robot ant moves from (0, 0) to (8, 12), then to (–5, 19) and back to (0, 0). It always takes the shortest path from one point to the next.Draw the ant’s path on 1-cm-grid paper and calculate:* the distance the ant travelled when moving from (–5, 19) to (0, 0)
* the total distance travelled by the ant.

Note: It’s a useful exercise to have students consider how to set up the axes for the task themselves, for example, using a portrait orientation with a one-to-one axis scale ratio, having the origin near the middle bottom of the graph paper, and labelling the origin and axes, with some values marked on the axes, for instance, –5, 0, 1, 5, 10, 15, 20.Check to see if students have any clarifying questions. |
| Body | Students complete the robot ant activity individually or in pairs. Use this time to observe students’ approaches and assess which students might need additional support or extension.Bring the class together as needed to discuss the robot ant task and students’ answers to the questions (slides 7–9):* the distance the ant travelled when moving from (–5, 19) to (0, 0)
* the total distance travelled by the ant.

It's important that students can calculate the distance between two points (the length of a line segment) before continuing to develop their sequence of steps. Provide additional practice as required.**Differentiation** (enable): Consider starting with two points in quadrant 1, so students can consolidate their understanding of the process and without needing to work with negative integers. Ensure students can draw in the corresponding right-angled triangle. The GeoGebra widgets listed under the Resources section may also be a helpful scaffold.* Do you think the hypotenuse will be longer or shorter than the total of the two side lengths combined? (Have students estimate the length of the hypotenuse by considering the length of the two side lengths.)
* If the side of each square on the grid is 1 unit in length (or imagine it’s the same as 1 step), how many units (steps) might the diagonal path from (–5, 19) to (0, 0) be? Why do you think that?
* [Concept of Cartesian coordinates – GeoGebra](https://www.geogebra.org/m/sh54rzkt) – this resource revises the location of points specified by coordinates on the Cartesian coordinate plane.
* [Length of Line Segments – GeoGebra](https://www.geogebra.org/m/Y6n2dAxB): this resource enables the construction of triangles using drag and place points and segments on a coordinate grid.

**Differentiation**: (extend): Consider having students explore the area of the triangle made by the ant robot. * Draw the smallest rectangle you can that encloses the triangle drawn by the ant’s path and has one of its sides along the *x*-axis.
* Can you use the area of the rectangle drawn to calculate the area of the triangle?
* [Pyth Thm: Coordinates – GeoGebra](https://www.geogebra.org/m/mbMVXfAM): this resource applies the theorem to two points in the plane and shows the relevant triangle and related calculations.
* [Length of a Line Segment & a Midpoint's coordinates on the Coordinate plane – GeoGebra](https://www.geogebra.org/m/temrdkjk): this resource applies the theorem to two points in the plane and shows the relevant triangle and related calculations across a broader range of values.
* [Coordinate Plane – GeoGebra](https://www.geogebra.org/m/VWN3g9rE): this resource enables various shapes to be drawn on a Cartesian coordinate plane.

**Developing a sequence of steps**At an appropriate time, have the class discuss points to consider when writing a set of steps. Ask students to share their thoughts (slide 12).* Be clear and simple: Imagine you’re explaining the steps to a friend who doesn't know much about the topic.
* Break it down into small steps: A bit like a recipe in cooking.
* Diagrams can be helpful.
* Review your instructions: Ask yourself whether someone who doesn't know the topic could follow your instructions.
* Test your steps out with someone else.

**Differentiation** (enable):Consider the make-up of a similar problem and observe each action to take. For example, what’s the first thing you would need to do? (Student: place 2 points on the graph.) That’s right, so what might be the first step to write down? (Student: draw 2 points anywhere on the graph paper.)So what would be the next step?* When writing the instructions, imagine you are giving the steps to a Year 6 student to follow – what might they need to know?
* Think about when and where including a diagram may be easier than trying to explain the instructions all in words.

**Differentiation** (extend):* See if you can write a set of steps to work out the area of any triangle drawn on the Cartesian plane.
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| Summary and reflection | Have students pair up and explain their sequence of steps to each other. Provide students with time to adjust their steps if necessary. Next, ask students to swap their instructions with each other and have the other person follow the instructions and provide constructive feedback. Consider developing a common and agreed whole-class sequence of steps. Sample steps with prompts:1. **Plot the points:** First, draw the two points on graph paper (digital or print). Label one Point A and the other Point B.
2. **Coordinates:** Write down the coordinates of Point A and Point B.
3. **Draw a right-angled triangle:** Imagine a right-angled triangle where Point A and Point B are two corners. The third corner is directly below or above (or horizontal to) either Point A or Point B. (You would want to see students have included a diagram(s) here*.*)
4. **Measure the sides:** Use the grids on the graph paper to calculate the lengths of the two sides that meet at the right angle. It doesn't matter if the coordinates of the points are negative, just think of it as a distance.
5. **Apply Pythagoras’ theorem:** Remember, the theorem is: $c^{2}= a^{2} + b^{2}$. Rearrange as follows to find the length of the hypotenuse: $c= \sqrt{(a^{2}+b^{2}})$.
6. **Write down your answer:** The square root you found is the exact distance between the two points on the graph. Remember to include the units of measure.
7. **Approximate the answer**: Use a calculator to approximate the answer to a suitable accuracy, for example, rounded correct to one decimal place. Remember to include the units of measure.

Bring the class together and have students share their reflections from the point of view of:* the writer of the steps
* a person following someone else’s steps.

An interesting point for discussion might be to explore how what seems clear and precise for one person may not be the same for another person. |
| Assessment | The following suggestion is given as an assessment opportunity.Exit ticket (slide 13)Question 1: What is the distance of the point (–7, 13) from the origin?Question 2: What is the length of the line segment connecting the points with coordinates (2, 8) and (10, 3)?Answers: $\sqrt{218}$ = 14.76 (correct to two decimal places); $\sqrt{89}$ = 9.43 (correct to two decimal places). |